Analysis of Effective Gadolinium Depletion Model

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UNIST

May 17, 2018
I. Introduction

II. Gadolinium Depletion Solvers

III. Gadolinium Depletion Results and Analysis

IV. Conclusions
STREAM/RAST-K Code System

- Flowchart of 2-step Approach

STREAM
2D Transport calculation
Group constants generation

RAST-K2.0
3D Diffusion Calculation
Core Simulation

STORA
Gathering STN files
XS data reformatting

Input files
*.inp

Output Files
*.out *.ppm

STN files
*.stn

XS table
*.XS

Input files
*.inp *.RI

Output Files
*.SUM *.RST
Case Matrix

**Full Case Matrix for Fuel**
- Burnup Steps (~80 MWd/kg)
- 42 burnup steps for FA without BA
- More burnup steps for FA with Gd
- 14 restart points
- \[ \Sigma = \Sigma_{base}(BU) + \Delta \Sigma_{TMO}(BU, TMO) + \Delta \Sigma_{BOR}(BU, BOR) + \Delta \Sigma_{TFU}(BU, TFU) + \Delta \Sigma_{CRD}(BU, TMO, BOR) \]

**Full Case Matrix for Reflector**

- \( T_f = T_m = 20 \)
- \( T_f = T_m = 425 \)
- \( T_f = T_m = 353 \)
- \( T_f = T_m = 293.6 \)
Depletion Modules in RAST-K 2.0

- **Microscopic Depletion**
  - Capability to track the number density and XS of major isotopes
  - Available to consider history effect for major isotopes

- **CRAM (Chebyshev Rational Approximation Method)**
  - Converged well even with shorter half life of isotope
  - Adopted for heavy nuclides chain and fission products chain

\[ \dot{N} = AN + b, \quad N = e^{At}N_0 + \left( e^{At} - I \right)A^{-1}b \]
\[ e^{At} = \alpha_0 I + 2 \text{Re} \left[ \sum_{i=1}^{k/2} \alpha_i \left( A - \theta_i I \right)^{-1} \right] \]
Microscopic Depletion Verification I

- **Heavy Nuclides Chain Result for SKN1 FA sets**
  - A0~C1C: FA w/o BA difference ~ ±20 pcm
  - B1~C1: FA w/ gadolinia difference ~ ±30 pcm
Microscopic Depletion Verification II

- **Heavy Nuclides + FPs Chain** Result for SKN1 FA sets
  - A0~C1C: FA w/o BA difference ~ ±40 pcm
  - B1~C1: FA w/ gadolinia difference ~ ±80pcm
I. Direct Numerical Solution of Linear Chain

\[
\frac{dN_m(t)}{dt} = N_{m-1}(t)\sigma_{m-1}(t)\varphi(t) - N_m(t)\sigma_m(t)\varphi(t), \quad (m = ^{154}\text{Gd} \text{ to } ^{158}\text{Gd}),
\]
where

\[N_m = \text{the number density of isotope } m, \text{ as a function of time } (t)\]
\[\sigma_m(t) = \text{the microscopic absorption cross section of isotope } m\]
\[\varphi(t) = \text{the neutron flux}\]

II. Effective Gd Isotope Depletion

These definitions depend only on the structure of burnup chain, not on cross sections

\[N_{\text{Gd}^{\text{eff}}} = 5N_{\text{Gd}^{154}} + 4N_{\text{Gd}^{155}} + 3N_{\text{Gd}^{156}} + 2N_{\text{Gd}^{157}} + N_{\text{Gd}^{158}}\]
\[\sigma_{\text{Gd}^{\text{eff}}} = \frac{\Sigma_{\text{Gd}^{154}} + \Sigma_{\text{Gd}^{155}} + \Sigma_{\text{Gd}^{156}} + \Sigma_{\text{Gd}^{157}} + \Sigma_{\text{Gd}^{158}}}{N_{\text{Gd}^{\text{eff}}}}\]
where
\[N_{\text{Gd}^{\text{eff}}} = \text{the effective number density}\]
\[\sigma_{\text{Gd}^{\text{eff}}} = \text{the effective microscopic cross section}\]
Simulation Code

- Tested Code
  - STREAM/RAST-K 2.0
  - Reference code: STREAM

- STREAM Calculation Conditions
  - 3-ring for Fuel pins, 10-ring for Gadolinia pins
  - Quadratic depletion for Gd isotopes
  - Critical spectrum OFF
  - Reflective B.C. for all directions

- RAST-K 2.0 Calculation Conditions
  - Microscopic Depletion
  - Eigenvalue search mode
  - Reflective B.C. for all directions
  - Transient Xe, Transient Sm
Test Model

- **16 × 16 Combustion Engineering (CE) type fuel assembly**
  - 850 K for fuel, 584 K for other materials
  - Boron concentration of 700 ppm

<table>
<thead>
<tr>
<th>Case</th>
<th>Fuel Pin ($^{235}$U wt.%)</th>
<th>Fuel Pins</th>
<th>Gadolinia Pin ($^{235}$U/Gd$_2$O$_3$ wt.%)</th>
<th>Gd Pins</th>
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- **35 Depletion Steps for Gadolinia Assembly**

```
0  0.1  0.5  1  1.5  2  2.5  3  3.5  4  4.5  5  5.5  6  6.5  7  7.5
8  8.5  9  9.5 10 11 12.5 15 17.5 20 22.5 25 27.5 30 32.5 35 37.5
40
```
Test Cases I Depletion Results – $k_{\text{inf}}$

**Case 1**
- Effective Gd ±20 pcm diff
- Direct Numerical Sol. ±60 pcm diff

**Case 2**
- Effective Gd ±30 pcm diff
- Direct Numerical Sol. >1000 pcm diff
Test Cases I Depletion Results – 1G Absorption XS

Case 1

Case 2
# Test Cases II

- **35 Depletion Steps for Gadolinia Assembly (Coarse)**

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</table>

- **70 Depletion Steps for Gadolinia Assembly (Fine)**

|   | 0  | 0.1 | 0.5 | 1  | 1.5 | 2  | 2.5 | 3  | 3.5 | 4  | 5  | 5.5 | 6  | 6.5 | 7  | 7.5 | 15 | 17.5| 20 | 22.5| 25 | 27.5| 30 | 32.5| 35 | 37.5| 40 |
|---|----|-----|-----|----|-----|----|-----|----|-----|----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|----|-----|
| 7.5| 8  | 8.5 | 9   | 9.5| 10  | 10.5| 11  | 11.5| 12  | 13  | 13  | 13.5| 14  | 14.5| 15  | 16  | 17  | 17.5| 18  | 18.5| 19  | 19.5| 20  | 21  | 21.5| 22  | 22.5| 23  | 24  | 25  | 25.5| 26  | 26.5| 27  | 27.5| 28  | 29  | 29.5| 30  | 30.5| 31  | 32  | 32.5| 35  | 37.5| 40  |
**Test Cases II Depletion Results – $k_{inf}$**

**Case 2-Coarse**
- Effective Gd $\pm 30$ pcm diff
- Direct Numerical Sol. $>1000$ pcm diff

**Case 2-Fine**
- Effective Gd $\pm 30$ pcm diff
- Direct Numerical Sol. $\pm 40$ pcm diff
Test Cases II Depletion Results – 1G Absorption XS

Case 2-Coarse

Case 2-Fine

[Graphs showing depletion results for Case 2-Coarse and Case 2-Fine]

May 17, 2018
Test Cases III

- 70 Depletion Steps for Gadolinia Assembly (Fine, Same)

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- Random step depletion calculation (Random)

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Test Cases III Depletion Results – $k_{inf}$

**Case 2-Fine, same**
- Effective Gd ±30 pcm diff
- Direct Numerical ±40 pcm diff

**Case 2-Random**
- Effective Gd ±30 pcm diff
- Direct Numerical Sol. ±180 pcm diff

<table>
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<th>R2-DNS</th>
<th>R2-effGd</th>
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<tr>
<td><strong>Time [sec]</strong></td>
<td><strong>Total</strong></td>
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<tr>
<td>1.942</td>
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<td><strong>Change [%]</strong></td>
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Conclusions

- Analysis about two kinds of Gd depletion models
- Direct Gd linear chain depletion
  - The direct Gd linear chain depletion model needs finer burnup steps to get acceptable accuracy
  - Individual isotope amount can be tracked
- Effective Gd depletion
  - The effective Gd depletion model provides accurate solution within 20 pcm due to the linear change of absorption cross section as a function of burnup
  - Simulation time is faster
- The effective Gd depletion model is suitable for RAST-K 2.0.

Future Works
- Quadratic Depletion as burnup
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