Uncertainty Quantification of LWRs by Random Sampling Method with STREAM/RAST-K

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1. Introduction

For the safety of nuclear power plant (NPP) operation, the accuracy of core simulation's result is very important. Now days, there are many powerful computing codes that can simulate the core quickly and reasonably using measured data. However, this codes usually do not include the effects of uncertainties in the input data used in the calculations. So, for the more precise predictions, the uncertainties in the data used should be considered.

In this paper, the important nuclear data in reactor core simulations are considered by assuming the variations in the input data follow the standard normal distribution. Because each nuclear data is not independent, the simulation should consider the relationship between them. For this purpose, there are random sampling method that use covariance of nuclear data to reflect above relationship and make perturbed sets. By using this varied data sets, core simulations for commercial LWR are carried out many times and search for the uncertainties of core properties are performed.

2. Method and

In this section, the procedure of how to vary the nuclear data by applying covariance data is described [1,2]. Also, the practical reactor model is briefly introduced.

2.1. Covariance data matrix

Covariance show the relation between two nuclear data. In this paper, the multi-group covariance data generated by the NJOY code are treated as one covariance matrix to consider the whole covariance data as symmetric matrix in one step.

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{a,a} & \mathbf{C}_{a,b} & \cdots & \cdots & \mathbf{C}_{a,z} \\ \mathbf{C}_{b,a} & \mathbf{C}_{b,b} & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \mathbf{C}_{y,y} & \mathbf{C}_{y,z} \\ \mathbf{C}_{z,a} & \cdots\vdots & \cdots & \mathbf{C}_{z,y} & \mathbf{C}_{z,z} \end{bmatrix}. \quad (1)$$

In this covariance matrix, the element, $C_{a,a}$ represent multi-group covariance between nuclear data expressed by its subscripts. If there are no covariance between two nuclear data, then that covariance element can be zero matrix.

This paper considers 4-types of nuclear data which are scattering, fission, capture, and nu-bar. For the cross-section data, the covariance data are generated by NJOY 99 with ENDF/B-VII.1 and the nu-bar covariance data are generated by NJOY 2012 with the same library [3].

2.2. Random sampling for nuclear data

For the variable, z that obeys standard normal distribution, the normal distribution that have α, β^2 as its mean and variance can be generated by

$$x = \beta \times z + \alpha. \quad (2)$$

Likewise, for the multivariate distribution vector **z** that include z following standard normal distribution as element, the multivariate distribution **x** can be generated by

$$\mathbf{x} = \mathbf{Az} + \boldsymbol{\mu}, \quad (3)$$

where $\boldsymbol{\mu}$ is mean vector of **x** and **A** is square matrix that satisfy the Eq. (4) [1].

$$\mathbf{C} = \mathbf{AA}^{\mathbf{T}}. \quad (4)$$

The **A** matrix can be solved by singular value decomposition of symmetric matrix $\mathbf{C}$.

$$\mathbf{C} = \mathbf{U\Sigma U}^{\mathbf{T}}, \quad (5)$$

where **U** is a set of orthonormal eigenvectors of $\mathbf{CC}^{\mathbf{T}}$**, which** means that $\mathbf{C}^2$ and $\boldsymbol{\Sigma}$ are diagonal matrix whose elements are also square roots of non-zero eigenvalues of $\mathbf{C}^2$. So, the **A** matrix can be expressed as in Eq. (6) [1,2].

$$\mathbf{C} = \mathbf{U}\sqrt{\boldsymbol{\Sigma}}\sqrt{\boldsymbol{\Sigma}}\mathbf{U}^{\mathbf{T}} = (\mathbf{U}\sqrt{\boldsymbol{\Sigma}})(\mathbf{U}\sqrt{\boldsymbol{\Sigma}})^{\mathbf{T}}, \quad (6)$$

$$\therefore \mathbf{A} = \mathbf{U}\sqrt{\boldsymbol{\Sigma}}. \quad (7)$$

Therefore, the random sampled set, **x** is

$$\mathbf{x} = \mathbf{U}\sqrt{\boldsymbol{\Sigma}}\mathbf{z} + \boldsymbol{\mu}, \quad (8)$$

2.3. Nuclear data library perturbation

In this study, nuclear data are perturbed with relative covariance matrix (absolute covariance matrix divided by expected value of nuclear data) given as [3]

$$\mathrm{rcov}(x,y) \equiv \frac{\mathrm{cov}(x,y)}{x_0 y_0}, \quad (9)$$

where

$$x_0 \equiv \mathrm{E[x]}, \text{ and } y_0 \equiv \mathrm{E[y]}.$$

Here, x, y mean probability distribution and E is expectation operator.

So, from above equation (8), the elements of set **x** can be expressed simply.

$$x_{t,g} = (\mathbf{U}\sqrt{\Sigma}\mathbf{z} + \boldsymbol{\mu})_{t,g} \times \sigma_{t,g}, \qquad (10)$$

where

x is perturbed elements of set $\mathbf{x}$,
σ is unperturbed elements of nuclear data set,
$\boldsymbol{\mu}$ is vector whose elements are 1,
t is type of nuclear data,
and
g is group of nuclear data.

2.4. Sampled nuclides

For the simulation of light water reactor, there arenuclides that largely affect to core calculation result. This kind of nuclides are already researched in other papers [4]. In this present paper, the 28 nuclides shown table 1 are considered by using ENDF/B-VII.1.

Table1 The list of most important nuclide in LWR calculation [4].

H-1	B-10	B-11	O-16	Zr-91
ZR-96	Rh-103	Xe-135	Sm-149	Gd-155
Gd-157	U-234	U-235	U-236	U-237
U-238	Np-237	Np-239	Pu-238	Pu-240
Pu-241	Pu-242	Am-241	Am-242*	Am-243
Cm-242	Cm-244	Cm-245		

2.5 Core model

A commercial core design of the APR-1400 and fuel model PLUS7 is used in this research. Specifically, the first cycle that has 241 feed fuel assemblies with different enrichment and number of gadolinia pin is analyzed.

	J	K	L	M	N	P	R	S	L
9	A	A	C	A	B	A	B	C	B
10	A	B	A	B	A	B	A	B	C
11	C	A	C	A	C	A	C	B	B
12	A	B	A	B	A	B	A	B	C
13	B	A	C	A	C	A	B	C	
14	A	B	A	B	A	B	C	C	
15	B	A	C	A	B	C	C		
16	C	B	B	B	C	C			
17	B	C	B	C					

Fig.1 The APR-1400 FA pattern

3. Numerical result

In this part, important core properties are calculated to obtain the mean and uncertainty. Totally, there are 500 samples and each of them is perturbed by random sampling method. From this perturbed nuclear data sets, 500 calculations are performed for the same model and the mean value and 1σ for some core properties are evaluated as uncertainty.

For the core simulation, STREAM, STORA, and RAST-K 2.0 are used and briefly described [5].

STREAM is a 2D lattice code that solves the neutron transport equation by the method of characteristics (MOC). STREAM calculates 2-group cross section data from perturbed nuclear data sets.

The STORA code have a role to connect STREAM and RAST-K 2.0 by gathering STN files that contains STREAM results which will be used by RAST-K.

RAST-K simulate whole core models by using 3D 2-group unified nodal method (UNM).

3.1. Critical boron concentration

The critical boron concentration (CBC) is calculated500 times. Fig.2 shows the samples mean and its standard deviation (1σ).

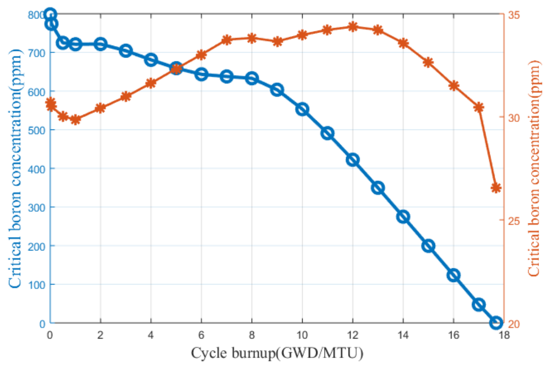

Fig.2 Critical boron concentration and its uncertainty (1σ).

The uncertainty CBC show some trend by following the amount of U-235, boron and trans-uranium nuclides change that have big role in core uncertainty analysis.

At the beginning of cycle (BOC), the amount of U-235 and boron decrease. It makes the decrease of uncertainty because they strongly affect to core uncertainty. However, the uncertainty starts to increase until some cycle burn-up point because of the effect of trans-uranium nuclides generation that act positively on the uncertainty exceeds the effect of decrease of U-235 and boron. However, there are one more turning point for uncertainty. It seems like mainly the effect of boron decrease that make uncertainty also decrease.

3.2. Axial power distribution

The axial power distribution and the uncertainty at BOC and EOC are shown in Fig.3.

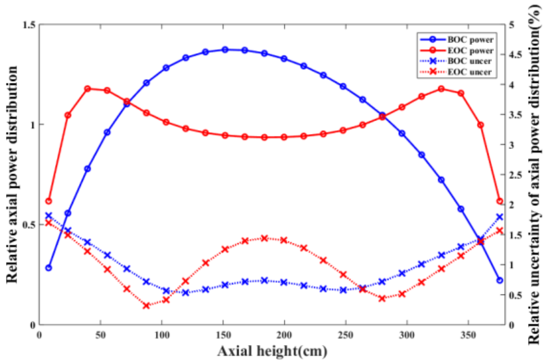

Fig.3 Relative axial power distribution and its relative uncertainty at EOC.

The interesting point that can be done by comparing two graphs is there are increasing trend for middle part of core. This trend is obvious because during the cycle burn-up, the fission happened in mainly core middle part, and there are more trans-uranium nuclides generation causing more drastic curve in uncertainty trend.

3.3. Radial distribution

For the radial power and burn-up distributions, theuncertainty is calculated as relative uncertainty (%). Each of these show similar trend at BOC and EOC. Identical to the CBC case, at BOC, the uncertainty value for whole region show higher value than at EOC.

	J	K	L	M	N	P	R	S	L
9	0.94								
	1.54								
10	0.9	1							
	1.45	1.31							
11	1.08	0.9	1.11						
	1.16	1.22	0.99						
12	0.92	1.04	0.92	1.04					
	1.05	0.97	0.93	0.72					
13	1.15	0.96	1.13	0.93	1.14				
	0.66	0.73	0.57	0.52	0.17				
14	0.98	1.17	0.96	1.06	0.94	1.02			
	0.44	0.32	0.33	0.14	0.09	0.5			
15	1.06	0.96	1.13	0.95	1.12	1.05	0.81		
	0.13	0.05	0.2	0.28	0.66	0.9	0.81		
16	1.11	1.03	1.06	0.99	1.07	0.78			
	0.66	0.64	0.7	0.74	0.9	0.87			
17	0.99	1.05	0.92	0.74					
	0.93	0.96	0.93	0.77					

FA POWER
Rel.Uncer(%)

Fig.4 Normalized FA power and its relative uncertainty (%) at BOC.

	J	K	L	M	N	P	R	S	L
9	0.05								
	1.5								
10	0.05	0.05							
	1.41	1.27							
11	0.05	0.05	0.06						
	1.13	1.19	0.96						
12	0.05	0.05	0.05	0.05					
	1.03	0.95	0.91	0.71					
13	0.06	0.05	0.06	0.05	0.06				
	0.65	0.72	0.56	0.52	0.17				
14	0.05	0.06	0.05	0.05	0.05	0.05			
	0.438	0.31	0.33	0.15	0.09	0.48			
15	0.05	0.05	0.06	0.05	0.06	0.05	0.04		
	0.13	0.05	0.19	0.27	0.65	0.88	0.79		
16	0.06	0.05	0.05	0.05	0.05	0.04			
	0.65	0.62	0.69	0.73	0.89	0.86			
17	0.05	0.05	0.05	0.04					
	0.92	0.95	0.92	0.76					

FA BU
Rel.Uncer(%)

Fig.5 Burn-up distribution and its relative uncertainty (%) at BOC. In this, BOC mean initial burn-up, 0.0500 GWd/MTU.

At the BOC, the uncertainty of power and burn-up show very similar value and trend. The reason for this is that the random sampling for the 28 nuclides is enough to describe the core at the BOC. However, for the EOC case, it shows very different result because there are many nuclide in the core at the EOC state which are not considered in random sampling.

	J	K	L	M	N	P	R	S	L
9	0.89								
	0.28								
10	0.92	1.1							
	0.29	0.32							
11	1.16	0.95	1.18						
	0.32	0.25	0.27						
12	0.94	1.12	0.96	1.13					
	0.2	0.21	0.18	0.14					
13	1.1	0.94	1.17	0.97	1.2				
	0.16	0.19	0.14	0.19	0.11				
14	0.94	1.12	0.96	1.14	0.97	1.14			
	0.24	0.16	0.21	0.12	0.19	0.14			
15	1.15	0.97	1.19	0.95	1.1	1.07	0.77		
	0.16	0.21	0.12	0.18	0.12	0.25	0.39		
16	1.18	1.11	1.09	0.99	0.91	0.69			
	0.16	0.13	0.12	0.16	0.26	0.43			
17	0.83	0.86	0.78	0.66					
	0.21	0.22	0.26	0.45					

FA POWER
Rel.Uncer(%)

Fig.6 Normalized FA power and its relative uncertainty (%) at EOC.

	J	K	L	M	N	P	R	S	L
9	16.15								
	1.26								
10	16.40	19.43							
	1.27	1.26							
11	20.65	16.97	21.25						
	1.24	1.25	1.22						
12	17.05	20.01	17.18	20.10					
	1.24	1.23	1.24	1.23					
13	20.51	17.22	21.05	17.13	21.13				
	1.17	1.21	1.21	1.22	1.21				
14	17.26	20.62	17.18	19.81	16.78	18.94			
	1.189	1.16	1.2	1.22	1.21	1.29			
15	20.14	17.24	20.71	16.48	18.97	17.81	12.86		
	1.22	1.2	1.22	1.2	1.24	1.34	1.29		
16	20.35	19.11	18.71	16.74	16.03	11.71			
	1.28	1.28	1.27	1.27	1.22	1.26			
17	15.01	15.67	13.80	11.32					
	1.2	1.2	1.22	1.24					

FA BU
Rel.Uncer(%)

Fig.7 Burn-up distribution and its relative uncertainty (%) at EOC.

3.4. Rod worth

APR-1400 have 7 control rod group that include two shutdown group (A, B). In this, the rod worth is analyzed as group worth for the BOC and EOC state with HFP, Eq. Xe condition.

Table2 Group worth and its relative uncertainty (%).

Group	BOC HFP, Eq. Xe	
	Group worth±1σ (pcm)	Relative Uncertainty (%)
5	300±3.72	1.24
4	405±2.80	0.69
3	760±9.77	1.28
2	977±7.66	0.78
1	1436±26.51	1.84
A	4640±27.68	0.59
B	5195±22.29	0.42

Table3 Group worth and its relative uncertainty (%).

Group	EOC HFP, Eq. Xe	
	Group worth±1σ (pcm)	Relative Uncertainty (%)
5	352±2.55	0.72
4	459±2.82	0.61
3	774±4.86	0.62
2	1019±5.69	0.55
1	1425±13.36	0.93
A	4174±35.60	0.85
B	4549±59.62	1.3

The whole rod worth results show the uncertainty less than 2%. However, there is new trend that the uncertainty of BOC case is higher than EOC case which are reverse result trend before this. The reason for this is that at BOC, there are smaller number of neutron than EOC, and if the neutron is absorbed by poison material, it causes more bigger fluctuation, meaning bigger uncertainty at BOC state.

5. Conclusion

This paper calculates the uncertainty of commercial LWR by using random sampling method. Many core properties are researched. However, there are other properties of core that uncertainty can be calculated. For future work, whole of them will be considered. Another point for improving this study is including many nuclides and its nuclear data. Unfortunately, the one of the most important nuclear data, fission spectrum, χ, are not considered in this paper. So, including χ and as many as possible nuclides will be key point not only for this random sampling method, but also other uncertainty quantification study.

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